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A review on the use of price index in national accounts

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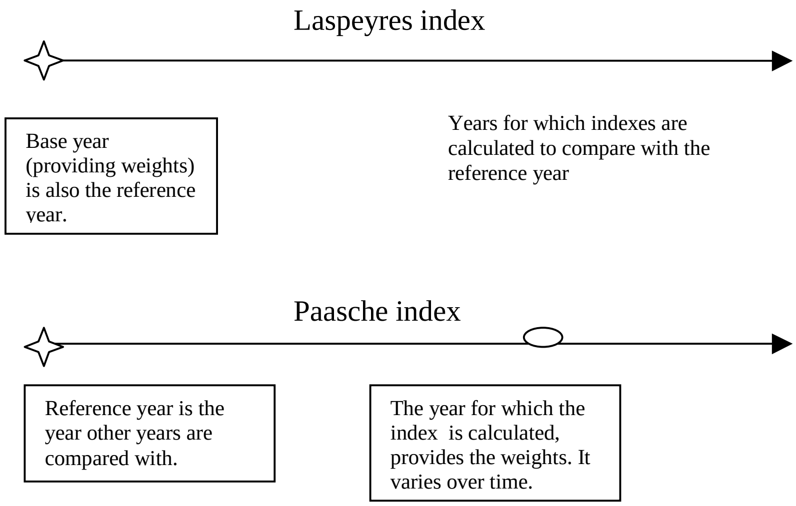
This review aims at further clarifying chapter XVI, "Price and Volume Measures", of the 1993 SNA with respect to impact on rates of changes and data requirement when different types of indexes are used in national accounts. Index is only a serious issue to study when it is applied to track changes in volume or price of a group of goods and services, which is an abstract concept with no explicit quantity or price. The review will discuss in part I the Laspeyres, Paasche and Fisher indexes as direct indexes when the base year is fixed. Part II covers the problems of rewriting history (rates of changes in prices and volume change) created by shifting base year periodically. Part III covers the current solutions to the problem of periodic shifting of base year: linking series with different base year or chaining of indexes which are annually rebased. Part IV discusses specific issues of national accounts.

I. Direct volume and price indexes

Direct indexes are calculated to compare two points in time. It is essentially a binary index where an index of a given point is calculated in comparison to another point. This type of binary index disregards the information of the intervening periods between the two points. The Laspeyres index uses a previous period to serve as the *base*, i.e. it uses their prices or quantities as weights. To each Laspeyres index, a correspondent index is attached, which is called Paasche index; it uses prices and quantities of latter period as weights or as the base period. The Fisher index is the geometric mean of these two indexes.

Figure 1: Illustration of Laspeyres and Paasche index



But indexes are not to be calculated to compare any pair of years only, analysts want indexes to depict real volume and price changes over time. As a consequence, for a time series, a *reference year* must be selected so that other years can be compared with it to derive the indexes. For direct indexes, specifically the Laspeyres, the calculation is still done in a binary way, i.e. an index for any given year is calculated to compare it with the year whose prices and quantities are used as weights. In this case, the year that provides the weights and the point of reference is called the *base year*. Individual indexes are still calculated without taking into account data in the intervening years.

In a time series, corresponding to each of this Laspeyres index is a Paasche index. So even though prices and quantities of the current year are used as weights in a Paasche index, implicitly the base for the comparison is still the *base year* of the associated Laspeyres index. A direct Fisher index as the geometric mean of the other two indexes therefore has to be based on the same initial base year.

Given a series of indexes with a base year, that base year is initially also used as the reference year, but later for convenience, another *Reference year* can be selected for the series. It is the year selected to have the index of 100. Indexes of other years can be mechanically grossed up or down proportionally to the original series of indexes, but its *base year* does not change.

Due to the close interrelationships between the Laspeyres, Paasche and Fisher indexes, when a new base year is selected for the Laspeyres indexes, the corresponding Paasche and Fisher indexes also change. These direct indexes will be explored in detail below.

1. Laspeyres index

The Laspeyres volume index uses prices of a given base year as fixed weights. The volume index of year t showing the growth from year 0 and year t, uses prices at year 0 as weights. The price index uses quantities of year 0 as weights. Formulas are as follow where the sum is over all elements in the group and the base weights are in parenthesis, q stands for quantity and p stands for price:

(1) $L_{qt}(p_0) = 3p_0q_t / 3p_0q_0$ (Laspeyres volume index)[1]

(2) $L_{pt}(q_0) = 3p_tq_0 / 3p_tq_0$ (Laspeyres price index)

2. Paasche index

The Paasche volume index uses prices of the current year t as fixed weights. The volume index of year t showing the growth from year 0 and year t uses prices at year t as

[1] The Laspeyres quantity index can be rewritten as follows:
$L_{qt}(p_0) = 3p_0q_t / 3p_0q_0 = (3p_0q_{0\ast}(q_t/q_0)) / 3p_0q_0 = (3v_0(q_t/q_0)$
Where v_0 is the base year value, q_t/q_0 can be replaced by any proxy index if quantities of period t are not available.

weights. The price index uses quantities of year t as weights. Formulas are as follow where the sum is over all elements in the group and the base weights are in parenthesis:

(3) $P_{qt}(p_t)= 3p_tq_t /3p_tq_0$ (Paasche quantity index)

(4) $P_{pt}(q_1)= 3p_tq_t /3p_0q_t$ (Paasche price index)

3. Relationship between Laspeyres and Paasche indexes

Indexes are used to measure volume changes (i.e. change in value at constant prices) and price changes over time. Volume in constant prices can be obtained by multiplying the base year value to the quantity index or by deflating value in current prices. The relationships between Laspeyres and Paasche indexes can be better seen in this light.

a) *Laspeyres value at constant prices of the base year is obtainable by deflating current value with its associated Paasche price index.*[2]

b) *Laspyes value at constant prices is also obtainable by extrapolating the base year value with its Laspeyres volume index.*[3]

c) *For any given year, the Laspyres volume index x Paasche price index = Paasche volume index x Laspeyres price index.*

d) *The most commonly used types of indexes in statistics until recently are the Laspeyres volume indexes and their implicit Paasche price indexes (property a). A common practice has been as follows: first, Laspeyres volume indexes are computed; second, values at constant prices are then derived by extrapolation (property of b); third, Paasche price indexes (implicit) are indirectly derived by dividing values at current prices with values at constant prices (property a).*

These relationships are shown or can be checked in the example given in table 1, which is calculated from raw data in table A in the appendix.

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[2] The value of a group of products at time t at constant prices at base year 0 is the sum of quantities at time t multiplied by their prices at the base year 0:

$$V_{t,co} = 3p_0q_t$$

Multiplying and dividing the right-hand side by the same current value at time t ($V_{t,cu} = 3p_tq_t$) will not change the value of the left-hand side. The result is shown below. Now the Laspyeres constant value equals the current value deflated by the Paasche price index $P_{pt} = (3p_tq_t / 3p_0q_t)$.

$$V_{t,co} = 3p_0q_t = 3p_tq_t / (3p_tq_t / 3p_0q_t)$$
$$= V_{t,cu} / P_{pt}$$

[3] The value at constant value at time t ($V_{t,co =}3p_0q_t$) can be rewritten by multiplying and dividing the right-hand side by the same value at the base year $V_0 = 3p_0q_{0:}$

$$V_{t,co} = 3p_0q_t = 3p_0q_0 * (3p_0q_t / 3p_0q_0) = V_0 * L_{qt}$$

Table 1. Relationship between Laspeyres and Paasche indexes

	Year 20 compared to year 0	
	Volume index	Price index
Base year 0 (Laspyeres)	3.230769	1.06153846
Base year 20 (Paasche)	2.521739	0.82857143
Current value of year 0	65	
Current value of year 20	174	
Calculating constant value and price index of year 20, base year = year 0		
Constant value of year 20 by extrapolating	Current value of base year*Laspeyres volume index = 65*3.230769 = 210	
Price index (Paasche)	Current value/constant value = 210/174 = 0.82857143	

Data sources: see table A, appendix.

4. Fisher index

The Fisher ideal index is the geometric mean of the Laspeyres and Paasche indexes:

$$(5) \qquad F_{qt} = (Lqt)^{1/2} \times (Pqt)^{1/2}$$

The Fisher index is called an ideal index as it satisfies the following tests:

- *Time reversal*: the time reversal requires that the index for period t based on 0 should be the reciprocal of that for 0 based on t;

- *Factor reversal*: the factor reversal requires that the product of the price index and volume index should be equal to the proportional change in the current values.

Laspeyres and Paasche indexes do not satisfy either of the tests. (The time reversal is clearly violated because the Paasche index is the reverse of the Laspeyres index but are not the same; the violation of the factor reversal is reflected in the fact that one cannot obtain constant value by deflating current value using the price index of the same type). Besides that, the Laspeyres and Paasche indexes do not reflect accurately the rates of growth in volume when the periods under examination are far away from the base year.

Different types of volume indexes and resulting volume growth rates are calculated and shown in table 2. Data used for the calculation are taken from table A in the appendix.

Table 2. Volume indexes and growth rates by different types of indexes

	Year 0,0	Year 0,10	Year 0,15	Year 0,20
Laspeyres				
Index	1	1.2923	2.6154	3.2308
Growth rate over previous period		29.23%	102.38%	23.53%
Paasche				
Index	1	1.2938	2.3200	2.5217
Growth rate over previous period		29.38%	79.32%	8.70%
Fisher				
Index	1	1.2930	2.4633	2.8543
Growth rate over previous period		29.30%	90.50%	15.87%

Data sources: see table A, appendix.

The indexes in table 2 are from year 0 to year 10, year 15 and year 20 respectively. Growth rates over previous year are calculated using those indexes. The following observations can be drawn from table 2:

a) *Laspeyres volume indexes are mostly higher than those of Paashce indexes*. This is true only if there is substitution effect in the economy, i.e. the decrease in relative price of a product shifts expenditures to another product. This is an important rule in economic theory. In our example (table A in the appendix), we assume that there are high-tech and non high-tech goods, it happens that prices of high-tech goods decline relatively to those of non high-tech goods and a higher share of expenditures is diverted to high-tech goods, except for year 10. This assumption is reasonable, as lower relative prices do not necessarily lead to substitution because either substitution takes time to effect or technical requirement does not allow substitution. For this reason, Laspeyres volume index in year 10 is lower than Paasche index.

b) *Fisher volume indexes are always in between those of Laspeyes and Paasche indexes.* Thus as compared to the Fisher indexes, Laspeyres indexes tend to provide higher volume growth rates for the current year and years close to the current year and Paasche indexes tend to provide lower growth rates for the current year, taking into account the clarification in (a).

c) *Paasche price indexes tend to overshoot the real changes in prices*. This observation is important in political and economic decision making as wages, transfer incomes tend to be adjusted automatically in many countries by changes in consumer price indexes.

Table 3. Comparison between Fisher index and Laspyeres index: volume in base year prices
(Base year: year 0)

	Year 0	Year 10	Year 15	Year 20
Laspeyres				
1. Non high-tech	45	60	90	90
2. High-tech	20	24	80	120
3. Total (1+2)	65	84	170	210
4. olume index	1.0	1.2923	2.6154	3.2308
5. Volume total extrapolated by index[4]	65	84	170	210
Fisher				
1. Non high-tech	45	60	90	90
2. High-tech	20	24	80	120
3. Total (1+2)	65	84	170	210
4. Volume index	1.0	1.2930	2.4633	2.8543
5. Volume total extrapolated by index	65	84.05	160.1	185.5

Data sources: see table A, appendix.

Additivity problem: For the above-mentioned crucial reasons in section 4, the SNA has recommended the use of the Fisher index. However, the Fisher index has one problem that is not faced by the Laspeyres and Paasche index, i.e. *The total volume derived by the volume Fisher index is not equal to the components at constant prices, especially for the years that are far away from the base year.* For years 15 and 20, volumes derived by the Fisher indexes through extrapolation, 160.1 and 185.5, are quite different from 170 and 210 respectively (see table 3). Laspeyres and similarly Paasche give the same totals.

II. Problems when base year changes

1. Effects of change in base year

As the base for an index series is far away from the current year, the structure in the components change. For example in our example in table A in the appendix, more high-tech goods are used as their prices decline. Laspeyres quantity indexes, which use prices of the base year, as weights tend to give higher weights to the current quantities, thus create higher rates of volume growth than they should be. The farther away the base year from the current period is, the larger the distortion is. Similarly, price indexes tend to be overestimated for the current year. For this reason, most countries change the base year every five years.

Table 4. Effects by base year changes: rates of volume growth over previous year (%)

Year	**Fisher index**			**Laspyeres index**		
	Base year 0	Base year 10	Base year 15	Base year 0	Base year 10	Base year 15
10	29.3			29.2	29.4	31.2
15	90.5	88.3		102.4	100.5	76.8
20	15.9	15.4	13.7	23.5	22.9	13.8

Data sources: see table A, appendix.

[4] Any difference in the total of the components and the total derived by using the indexes to extrapolate the base year value is due to rounding off error.

For illustration, table 4 shows changes in rates of volume growth when the base year changes. As the base year is closer to the current year, rates of growth are reduced. This is true for both the Laspereyrs and Fisher indexes. Table 4 also shows that rates of growth by Fisher indexes are normally lower than those of Fisher indexes, except year 10 when substitution effects do not take place as previously explained.

Changing the base year is like trying to rewrite history, as rates of changes in volume and prices change. A base year from the very far-away past will create unrealistically higher rate of changes for the recent years. In the USA, the 1987 fixed weights increase real GDP annual growth rate from 2.3 percent to 2.4 percent. On the contrary, a more recent base year will underestimate the rate of changes in the past. In the USA, with the 1987 fixed weights, real GDP annual growth for 1929-87 reduces from 3.4 percent to 3 percent and especially real GDP dropped 25 percent from 1944 to 1947, but with more appropriate weights, GDP declined by only 13 percent[5].

2. Linking series of different base years

Laspeyres volume indexes need to change the base year when the current year is too far away from the base year in order to reduce the rate of growth created by changes in relative prices and the consequent substitution effects. Whenever that happens, it is sometimes necessary to link (or chain) the two data series for analysis so as not to change the rates of growth in the past. The result is that GDP at constant values for previous series is not equal to the sum of the components at the new base constant prices.

Linking can be simply done by extrapolating backward the index of the new base year using the rates of change in the indexes of the old base year, assuming that the new base year is used as the *reference year* (which is normally the case). Figure 2 shows the linking procedure. The total values of the old series are calculated by using the new rebased indexes. The components of the old series which move with their own quantities and prices must be individually calculated using prices of the new base year in order not to destroy the structure of the expenditures. This linking creates a serious consequence: the re-based components do not add up to the new total. In table 5, as an example, for year 0, after linking, the sum of the components is 45 + 20 = 65, which is not the same as 64.07. If year 0 is used as the reference year, the index series of new base year 10 must be grossed down to the reference year, we have the same problem that the total is not equal to the sum of the components (the so-called *additivity* problem).

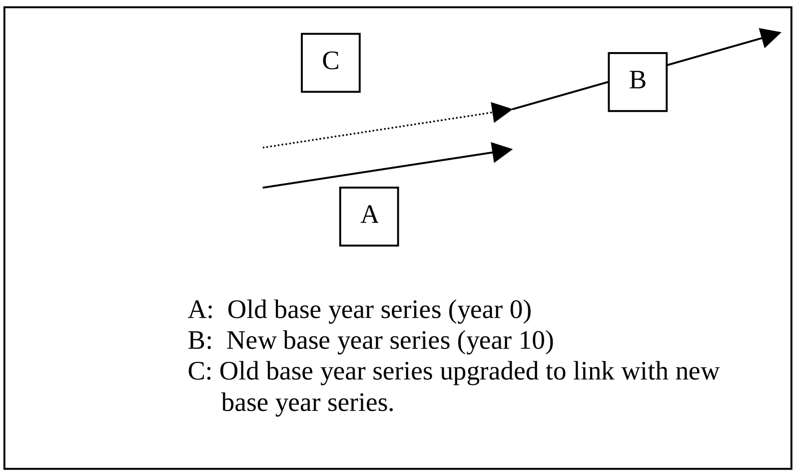
[5] "BEA's chain indexes, time series, and measures of long-term economic growth," J. Steven Landerfeld and Robert P. Parker, *Survey of Current Business*, May 1997.

Table 5: Example of linking data of the two base years

	Base: year 0		Base: year 10		
	Year 0	Year 10	Year 10	Year 15	Year 20
	Q_0P_0	$Q_{10}P_0$	$Q_{10}P_{10}$	$Q_{15}P_{10}$	$Q_{20}P_{10}$
Non-high-tech	45	60	60	90	90
High-tech	20	24	22.8	76	114
Total	65	84	82.8	166	204
Composite volume index	1.0000	1.2923	1.0000	2.0048	2.4638
	After linking to base year 10				
Non-high-tech	45		60	120	120
High-tech	19		22.8	36	54
Total	64.07		82.8	156	174
Composite volume index	0.7738		1.0000	2.0048	2.4638

Data sources: see table A, appendix.

Figure 2: Linking procedure



The change of base year and the linking of two or many series of different years is widely exercised. Rates of growth are kept the same as they are with the new base. Linking is therefore different from recalculating indexes for previous years using the new base year. The latter is the rewriting of history as growth rates change.

3. Chaining annually rebased indexes

Chaining indexes are possible only when the base year changes annually. An example in table 6 below shows the chaining of annual Fisher indexes though chaining can be done for either the Laspeyres, Paasche or Fisher indexes. Before chaining, theoretically Fisher every year, volume index must be calculated with a new base year. Again the clarify the issue, direct Fisher index and chain index are shown.

The direct Fisher index: This index is calculated to measure change between two years by taking into account prices and quantities of any two years 20 only (see formulas 1, 2, and 3). A direct volume index compares a terminal year, say year 20, to an initial year, say year 0. The index from year 0 to year 20 is 2.8543.

The linked Fisher index: The linked Fisher indexes, the series: 1.930, 1.8829, 1.1266 in bold face in table 6, are obtained by changing the base year annually. These linked indexes are now widely called the chain Fisher index. The actually chain index is a special case, which is described below. The linking or chaining of the data into a time series is simple. One can pick any year as the *reference year* with an index of 100, indexes of other years are then scaled to that year using the annual percentage change provided by the annual index.

The chain Fisher index: The chain index takes into consideration all information on the shifting weights of relative prices and quantities of the intervening years between the two years the index is aimed to compare. Let us call F(i,j) is the direct index from year i to year j, then the chain index (year 0 to 20) = F(0,10) x F(10,15) x F(15,20) = 1.2930 x 1.8829 x 1.1266 = 2.7428. This chain index is smaller than 2.8543, the direct index between the two years.

Table 6. Fisher's direct index and chain index

Terminal year	Initial year			
	Year 0	Year 10	Year 15	Year 20
Year 0	1.0000			
Year 10	**1.2930**	1.0000		
Year 15	2.4633	**1.8829**	1.0000	
Year 20	2.8543	2.1729	**1.1266**	1.0000

Data sources: see table A, appendix.

IV. Indexes in national accounts

From the discussions above, in theory data on prices, quantities for every year are for the calculation of either Laspeyres or Fisher indexes. So there is no reason why Fisher chain indexes are not used, even though it creates the problem of additivity. When data are available, it is preferable to use chain indexes of Fisher type as recommended by the SNA. The problem is that data in current year in many countries do not have the detailed data on quantity. For the more current years, countries are likely to have most data in values but few data on quantity. Detailed data on quantity are available only if the country carries out a census and supplemented every year with an annual survey to provide the same details. Annual surveys normally provide less detail; in that case, the weights in the base year (the census year) have to be used for aggregation and the resulting annual indexes are no longer pure, they are more like proxies. Previously it has been shown that the Laspeyres volume index can be written in terms of the base year values ($v_{0)}$ and the quantity ratios (see footnote 1), $L_{qt}(p_0) = (3v_0(q_t/q_0)$, it is possible to design some ratios of q_t/q_0 instead of having data on quantities for the current year. Below are discussions on few important aggregates:

1. GDP/Value added

GDP can be estimated by three methods: (1) the production approach; (2) the income approach; and (3) the final expenditure approach.

The production approach obtains the value added for each industry by deducting intermediate consumption from output and then summing up these value added to obtain GDP. To apply Laspeyres and especially Fisher formulas appropriately, one needs output of each industry by type of products and also intermediate consumption by type of products consumed by each industry for every year. To arrive at GDP at constant prices, one needs the get output at constant prices and intermediate consumption at constant prices in order to get value added by industry. (This is the double deflation method). Data required for this task is hardly met by any country and more so in developing countries, especially at the end of the current year. Some proxies or quantity indexes have to be created for the more current years as previously discussed. Deflating using this approach provides not only GDP but also value added by industry at constant prices. This approach is the most data demanding. Many developing countries simply use consumer price index (CPI) to deflate GDP.

The final expenditures approach sums up final consumption expenditures of households, government and non-profit institutions, gross capital formation, exports less imports to obtain GDP. This approach provides a less demanding way in calculating volume index for GDP because only final goods and services are required. It is now a practice in the Britain, the USA, Canada and other countries.

The income approach sums up components of value added to obtain GDP. Constant GDP has never been obtained by using this approach.

2. CPI

Consumer price index (CPI) is generated by dividing the values at current prices and at the base year prices of the same basket of goods and services purchased by households at the base year. It is an index widely used to determine wages in labor contracts, income transfer programs of government, etc. This is a Laspeyres price index. Because of that technically it cannot be use to deflate current values; only the Paasche price index should be used (see item a in section 3, part I.). With current weights, the Paashce CPI index can be developed. It reflects more the substitution effects, which is preferred by economists. It should also be mentioned that CPI is mostly developed for urban areas for the basket of goods and services urban residents purchased. It is not fully compatible with the SNA concept of final consumption expenditures of households as there are goods and services produced for own final consumption and other imputed expenditures by the SNA such as own-occupied housing. To broaden the CPI concept to rural areas and to make it compatible with the SNA is more data demanding since production for own final consumption in rural areas is much higher than in urban areas.

Appendix

Table A. The example used in the paper

	Year 0			Year 10			Year 15			Year 20		
	Quantity	Price	Value	Quantity	Price	Value	Quantity	Price	Value	Quantity	Price	Value
Non-high-tech	15	3	45	20	3	60	30	3.5	105	30	4	120
High-tech	5	4	20	6	3.8	22.8	20	2	40	30	1.8	54
Total			65			82.8			145			174
Structure												
Non high-tech			0.692			0.725			0.724			0.690
High-tech			0.308			0.275			0.276			0.310

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