PUBLIC INVESTMENT CRITERIA: ECONOMIC INTERNAL RATE OF RETURN AND EQUALIZING DISCOUNT RATE
by
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FOREWORD

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Burnham O. Campbell
Chief Economist
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I. INTRODUCTION

1. The economic analysis of a project is aimed at determining whether the project is consistent with the overall national and sectoral objectives and whether the investment proposed represents the best means of achieving the intended objectives. The process of economic analysis can be seen as a sequence of actions. First, it is necessary to identify the need or demand for the project. The second step is to establish whether the proposed project provides the least cost or the most cost-effective way of attaining the objectives of the project. Having identified the project's costs and benefits and carefully quantified them, the third step is to ascertain whether the net benefits expected from the resources allocated to the project would be in excess of, or at least equal to, the net benefits to the economy that could be expected if these resources were made available for the best alternative project. These three steps are distinct but interrelated.

2. The literature on public investment criteria has focussed attention on the third step whereby the net discounted present value and economic internal rate of return (EIRR) have emerged as the main criteria for judging the economic viability of a project. The purpose of this paper is to focus attention on the second step in which the least-cost solution is determined. Apart from highlighting the issues involved in deriving the least-cost solution, some insights about the linkages of the three steps in the process of economic analysis of projects will emerge.

3. The standard literature on the theory of the firm will be used to provide the analytical backbone of this paper. Starting from a simple static one-period model, we will move to a static multi-period framework which will provide the basis for the discussion of investment criteria that is to follow. In sections II and III the emphasis has been to clearly distinguish profit maximization from least-cost solutions, thereby providing the rationale for the distinction between EIRR and the equalizing discount rate (EDR). While the model described in section III is rarely used in determining the least-cost solution among project alternatives, its usefulness lies in allowing a rigorous examination of the issues involved. It provides the linkage between the three steps described above in the economic analysis of projects. A numerical example is given in order to clarify the issues involved.

II. STATIC ONE-PERIOD MODEL

4. Consider a public utility (power) in a static one-period framework. It faces a demand schedule:

\[ p = a - bq \]  

(1)

A linear demand schedule has been chosen for analytical simplicity. The results can be generalized to a non-linear demand schedule.
where $p$ represents price, $q$ quantity supplied, $a$ and $b$ are parameters of the demand schedule. The production function is given by

$$q = q(x_1, x_2)$$  \hspace{1cm} (2)

where $x_1$ and $x_2$ are substitutable inputs with given input prices $(c_1, c_2)$. The problem facing the public utility can be represented by

Maximize \hspace{1cm} \int_{0}^{q} (a - bq) \, dq - c_1 x_1 - c_2 x_2 \hspace{1cm} (3)

wrt $q$, $x_1$, $x_2$

subject to $q = q(x_1, x_2)$

However, if the output level of the public utility is fixed at $\bar{q}$ the problem is represented by

Maximize \hspace{1cm} \int_{0}^{\bar{q}} (a - bq) \, dq - c_1 x_1 - c_2 x_2 \hspace{1cm} (4)

wrt $x_1$, $x_2$

subject to $\bar{q} = q(x_1, x_2)$ \hspace{1cm} (5)

The above can be written alternatively as

Minimize \hspace{1cm} c_1 x_1 + c_2 x_2 \hspace{1cm} (6)

wrt $x_1$, $x_2$

subject to $\bar{q} = q(x_1, x_2)$

From the above it is clear that a profit maximizing problem is equivalent to a cost minimizing problem when the output level is fixed. Thus the cost minimizing problem is a subcase of the more general profit maximizing problem.

Having determined $(x_1^*, x_2^*)$, the optimal solution to the cost minimizing problem, these values can be substituted into (4) to see whether net benefit is positive or not. This net benefit will be less than or equal to the net benefit associated with the profit maximization problem of (3).
III. STATIC MULTI-PERIOD MODEL

7. It should be stressed that the formulation in Section II is sufficiently general to incorporate the case of a multi-period framework. In the latter case it must be interpreted as listing an input (output) of different periods as being different inputs (outputs). In the multi-period framework the problem is that of maximizing the net discounted present value of the stream of benefits for the profit maximizing problem, or minimizing the net discounted present value of the stream of costs for the cost minimizing case.

8. The above general framework can be used as an efficiency criterion for choosing among projects. For example, \( q_t \) can be interpreted as the output of the public utility in time \( t \), \( x_1 \) and \( x_2 \) as the alternative input streams for projects 1 and 2 respectively, and \( c_1 \) and \( c_2 \) are the costs of the inputs. The following integer programming problem could thus be posed.

Maximize

\[
\sum_{t=0}^{T} D_t \left[ (a-bq^t)q^t - d_1 c_1 x_1^t - d_2 c_2 x_2^t \right]
\]

wrt \( q_1, d_1, d_2 \) (7)

subject to \( q^t = q \left( d_1 x_1^t, d_2 x_2^t \right) \) (8)

\[
\sum_{i=1}^{2} d_i = 1
\]

\( d_i = 0, 1 \quad i = 1, 2, \) (9)

where \( D_t = 1/(1+r)^t \) is the discount factor in the period \( t \), \( r \) is the opportunity cost of capital, and \( d_i \) is the integer variable.

9. If the output level of the public utility is fixed at \( q^t \), \( t=0, \ldots, T \) then the problem reduces to

Minimize

\[
\sum_{t=0}^{T} D_t \left[ d_1 c_1 x_1^t + d_2 c_2 x_2^t \right]
\]

wrt \( d_1, d_2 \) (11)

subject to \( \sum_{i=1}^{2} d_i = 1 \) (12)

\[
\sum_{i=1}^{d_i} = 0, 1 \quad i = 1, 2,
\]
As in the static one-period model, having determined the minimum cost project from the integer programming exercise, the cost stream associated with the optimal solution can be substituted in (7) to see whether the discounted present value is positive or not.

10. The above general procedure provides the rationale for investment criteria adopted by the Bank in its Guidelines for Economic Analysis of Projects. The basic economic criterion for the acceptability of a project involves the satisfaction of two conditions. First, the present value of the net benefits of the project must not be negative. Second, the net present value of the project must be higher than, or at least as high as the net present value of mutually exclusive project alternatives. Since a systems approach suggested by the general integer programming model described in para 8 is not feasible in most cases, the determination of the least-cost solution and the condition that the net discounted present value is non-negative are used to satisfy the two conditions required for economic justification of a project.

11. Two important points emerge from this discussion. First, in the least-cost solution, mutually exclusive projects which provide alternative ways of producing exactly the same output are considered. Since benefits are the same in such cases, it is necessary only to consider costs and to select the alternative with the lowest present value of cost. An ex-ante choice between two mutually exclusive projects is then made. Second, this least-cost solution by itself says nothing about the economic merits of the project. Hence the cost stream of the least-cost solution is compared with the benefit stream to determine whether the discounted present value of the net benefit is positive. It is important to note that the benefit stream is measured by willingness to pay.

IV. THE LEAST-COST SOLUTION: A HEURISTIC APPROACH

12. In practice, the least-cost solution is rarely derived from an integer programming model. Instead, two or three technically feasible alternatives are considered and a minimum cost test is applied to determine the least-cost solution. The test consists of calculating the present worth of the cost streams associated with alternative project technologies, design standards, or phasing for a range of discount rates. An example is the comparison between a hydro and a thermal cost stream for implementing a power program. While the hydro project has higher initial capital costs, its operating costs are lower. The opposite is true for the thermal project. Consequently, the preference ranking of the hydro and thermal alternatives (indicated by the cost stream with the lowest present worth) may change between lower and higher discount rates. At low discount rates the present worth of the costs associated with the hydro project will be lower than that associated with the thermal project. The discount rate at which the preference changes is known as the cross over discount rate or the equalizing discount rate (EDR). In terms of the framework described in
para 9, the EDR ($d^*$) is defined as the discount rate which equalizes the two cost streams.

\[
\sum_{t=0}^{T} \frac{c_1 x_1}{(1+d)^t} = \sum_{t=0}^{T} \frac{c_2 x_2}{(1+d)^t}
\]  

(13)

Assume that $x_1$ refers to the thermal option and $x_2$ refers to the hydro option. If $d^*$ is greater than the discount rate, then the present worth of the hydro option is lower than that of the thermal option. Thus if the equalizing discount rate $d^*$ is higher than the opportunity cost of capital then the option with the higher initial costs will be the least-cost option (and vice versa). This is clarified in the example given below.

13. In Table 1 the cost streams of a hydro and thermal power stations are given. Assume that all of the capital expenditures are undertaken in year 0. The initial expenditure for the hydro project is higher, but it has lower operating costs throughout the life of the project. The computations for deriving the EDR are shown in Table 1. At a discount rate between 10 and 15 per cent, the least-cost option changes from the hydro project to the thermal project. Through a process of linear interpolation, the EDR at which the switchover occurs is estimated to be 13.1 per cent. The EDR can also be derived graphically as shown in Figure 1 where the present worth of each option is taken at 10 and 15 per cent and plotted. Assuming linearity, the switchover point represents the EDR.

14. If the opportunity cost of capital is 10 per cent, then as is clear from Figure 1, the hydro project with higher initial capital expenditure constitutes the least-cost option. At 10 per cent its net worth is lower. If the opportunity cost of capital is 15 per cent, then the thermal project with lower initial capital but higher operating costs and lower net worth constitutes the least-cost solution.

15. In practice, least-cost analysis is usually expressed in terms of the EDR between the costs of different options which deliver the same benefits. Calculation of the EDR involves a comparison between the two cost streams of the projects under consideration, but does not include the current source of supply. The alternatives considered must be realistic in the sense that in the absence of the proposed project, the alternative method would prevail. In other words, if the hydro option which is the cheapest is not chosen, the thermal option will be undertaken. It cannot be overemphasized that the EDR calculations rank mutually exclusive ex-ante options.
V. THE USE OF THE LEAST-COST SOLUTION IN ESTIMATING THE EIRR

16. Having determined the unique least-cost solution using the EDR technique, we compare the cost stream of the least-cost solution with the expected benefit stream. In terms of the formulation given in para 8 and assuming that project 1 represents the least-cost solution, we estimate

\[ \sum_{t=0}^{T} D_t \left[ \frac{-q^t}{a - b q^t} dq^t - c_1 t x_1^t \right] \]  

(14)

to find out if the net discounted present value of benefits is positive. The important point to note here is that once project 1 is deemed to constitute the least-cost solution, the costs associated with project 2 have no role to play in determining the economic viability of project 1.

17. The implicit assumption made thus far is that the project leads to an increase in supply which is valued in terms of willingness to pay. This is a simplifying assumption which is made so that attention can be focused on the analytics of the least-cost solution. The difference between the availabilities of inputs and outputs with and without the project constitutes the basic method of identifying the project's costs and benefits. It is possible that using the with- and without-project methodology that the benefit of the project (the hydropower plant) under consideration may consist of

(i) the diversion of supply from an existing source (diesel) to a more "efficient" alternative (hydro), total power output remaining constant,

(ii) additional supply of power for meeting previously suppressed demand, or

(iii) a combination of (i) and (ii).

18. In the identification and quantification of benefits the above must be considered. In case (i), the resource cost saving in switching the source of supply from the existing diesel plant to the proposed hydro alternative constitutes the benefit of the project. This resource cost saving is then used to estimate the net discounted present value of the benefit or the economic internal rate of return. In the without-project situation the existing diesel plant must continue to be used; hence, introduction of the proposed hydro plant constitutes a real resource cost saving to the economy.

19. The discussion in this section raises two important points. First, once the least-cost alternative is determined, the next best alternative has no role whatsoever to play in the EIRR analysis. Second, if the proposed project leads to substitution of a more expensive existing source of supply by the output of the proposed project with net additional supply being zero, then the real resource
cost saving generated by the introduction of the proposed project represents the benefit to be used in the EIRR analysis.

VI. CONCLUSIONS

20. Having determined the need for the project, the next step is to identify the least-cost means of achieving the desired end. This paper deals with this set of issues. The least-cost analysis is expressed in terms of the equalizing discount rate (EDR) between the cost of different options which deliver the same benefit. Calculation of the EDR involves a comparison between the two cost streams of the projects under consideration and does not include the current source of supply. The alternatives considered must be realistic in the sense that in the absence of the project being proposed, the alternative method would prevail.

21. However, by itself, least-cost analysis says nothing about the economic merits of the project since even a least-cost project may have costs that exceed its benefits. For example, a mini hydro maybe cheaper than its diesel alternative, but the cost of even a mini hydro may exceed the benefits. Hence, whenever possible, it is necessary to consider whether benefits are adequate. In particular, differences in costs as between the least-cost design and the next best alternative are not, and should not be used as a measure of benefits of such projects. The comparison between the value of benefits and costs incurred for realizing these benefits is expressed in terms of the economic internal rate of return (EIRR). Thus, while the EDR calculations rank mutually exclusive options, the EIRR calculations determine whether the costs of any one of the options are commensurate with benefits.
Table 1. Choice Between Hydro and Thermal Alternatives
Illustrating Equalizing Discount Rate

<table>
<thead>
<tr>
<th>Year</th>
<th>HYDRO POWER PROJECT</th>
<th>THERMAL POWER PROJECT</th>
<th>EQUALIZING DISCOUNT RATE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Cost Stream</td>
<td>Present Worth 10%</td>
<td>Present Worth 15%</td>
</tr>
<tr>
<td>0</td>
<td>900</td>
<td>900.0</td>
<td>900.0</td>
</tr>
<tr>
<td>1</td>
<td>165</td>
<td>150.0</td>
<td>143.5</td>
</tr>
<tr>
<td>2</td>
<td>165</td>
<td>136.4</td>
<td>124.8</td>
</tr>
<tr>
<td>3</td>
<td>165</td>
<td>124.0</td>
<td>108.5</td>
</tr>
<tr>
<td>4</td>
<td>165</td>
<td>112.7</td>
<td>94.3</td>
</tr>
<tr>
<td>5</td>
<td>165</td>
<td>102.5</td>
<td>82.0</td>
</tr>
<tr>
<td>6</td>
<td>165</td>
<td>93.1</td>
<td>71.3</td>
</tr>
<tr>
<td>7</td>
<td>165</td>
<td>84.7</td>
<td>62.0</td>
</tr>
<tr>
<td>8</td>
<td>165</td>
<td>77.0</td>
<td>53.9</td>
</tr>
<tr>
<td>Total</td>
<td>2220</td>
<td>1780.4</td>
<td>1640.3</td>
</tr>
</tbody>
</table>

Equalizing Discount Rate = 10 + 5(40 / 63.3) = 13.1%
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