ONLINE APPENDIX

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THE PATH TO KINA CONVERTIBILITY: AN ANALYSIS OF PAPUA NEW GUINEA'S FOREIGN EXCHANGE MARKET

Martin H. Davies and Marcel Schröder

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APPENDIX

A. Exchange Rate Expectations

We assume that agents' expectations of the equilibrium real exchange rate (ERER), e, and the equilibrium level of absorption, A, are jointly and uniformly distributed on the circle centered around point (A^*, e^*) with radius r, where:

$$f_{A,e}(A^*, e^*, r) = \begin{cases} \dfrac{1}{\pi r^2} & if\ (A - A^*)^2 + (e - e^*) \leq r^2 \\ & 0\ otherwise \end{cases}$$

From this, we calculate marginal distribution of the ERER, e, which is:[1]

$$f_e(e, e^*, r) = \frac{2\left(r^2 - (e - e^*)^2\right)^{\frac{1}{2}}}{\pi r^2} \quad (1)$$

It follows that the cumulative marginal distribution function is calculated as:

$$F_e(e_0, e^*, r) = \frac{2}{\pi r^2} \int_{-r}^{e_0} (r^2 - (e - e^*)^2)^{\frac{1}{2}}\, de \ \text{ for } e_0\ \epsilon [e^* - r, e^* + r]$$

This simplifies to:

$$F_e(e_0, e^*, r) = \frac{1}{2} + \frac{1}{\pi} \begin{bmatrix} sin^{-1}\left(\frac{e_0 - e^*}{r}\right) + \\ \left(\frac{e_0 - e^*}{r}\right)\left(1 - \left(\frac{e_0 - e^*}{r}\right)^2\right)^{\frac{1}{2}} \end{bmatrix} \text{ for } e_0\ \epsilon [e^* - r, e^* + r]$$

Given this, then $F_e(e^* - r, e^*, r) = 0, F_e(e^*, e^*, r) = \frac{1}{2}$ and $F_e(e^* + r, e^*, r) = 1$.

The assumption of uniformity of the joint distribution is made in the interest of analytical tractability. However, as we can see from the marginal distribution of the ERER, (equation 1); this imposes that the distribution of expectations for e is concentrated around the central value, e^*, and is increasing as approached from above or below e^*. We assume further that each agent, whether a stockholder of foreign exchange (forex) or kina, has an equal share of that particular stock. These assumptions mean that the functions $\Upsilon(e_0, e^*, r, \sigma_0, \delta)$ and $\tau(e_0, e^*, r, \Delta i, \sigma_1)$, which represent the fraction of the stock brought to the market for kina stockholders and forex stockholders, respectively, may be calculated using the cumulative marginal density for exchange rate expectations accounting for the behavior of each group of stockholders.

B. Demand for Foreign Exchange

Decision of holders of kina. Stockholders of kina—primarily domestic firms that seek to access forex to buy foreign goods, repatriate profits/dividends, or repay forex loans—have a different set of

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[1] This is calculated by integrating $f_{A,e}(e^*, A^*, r)$ over $A\ \epsilon\ [A^* - r, A^* + r]$, that is $f_e(e, e^*, r) = \int_{A^*-r}^{A^*+r} f_{A,e}(e^*, A^*, r)\, dA$.

incentives compared to the holders of the forex stock.[2] Whenever an agent decides to convert kina to forex in the absence of full convertibility, they must negotiate and queue to purchase forex and so incur shoe-leather costs, σ_0, which are modelled as iceberg costs.

However, it is costly for agents to delay the forex transactions they seek to execute because they are impatient and may incur additional pecuniary and non-pecuniary costs; for example, loss of reputation with their foreign parent companies for nonpayment of dividends and loans, loss of domestic sales opportunities if they cannot deliver goods on time to the domestic market, and penalties imposed by foreign lenders for late or deferred payments on loans. We model these costs using the factor δ.

In this case, domestic firms must compare the costs of converting kina to forex now versus waiting until the next period. Given the exchange rate, e_0, the desire to convert K_D kina, and the need to queue and incur shoe-leather costs, then this will convert into F_D^o units of forex, where:

$$F_D^0 = \frac{K_D}{e_0(1+\sigma_0)}$$

If they delay the transaction and convert in the next period when the exchange rate has adjusted to e_1^e (the agent's expected exchange rate), then K_D is discounted by factor $(1 + \delta)$, but there are no shoe-leather costs because the expected exchange rate, e_1^e, ensures currency convertibility. Then:

$$F_D^1 = \frac{K_D}{e_1^e(1+\delta)}$$

The break-even point occurs when $F_D^1 = F_D^0$ which gives us:

$$\frac{(1+\sigma_0)}{(1+\delta)} = \frac{e_1^e}{e_0}$$

which may be rearranged to give a parity condition relating the expected depreciation to the difference between shoe-leather costs and the costs of delay:

$$\frac{\Delta e^e}{e_0} = \sigma_0 - \delta > 0$$

Intuitively, this equation states that an agent is indifferent between conversion now or later when the expected depreciation is just offset by the difference between the shoe-leather costs and the costs of delay. This may be solved for the break-even spot rate, e^d, which depends on an agent's expected future spot rate, e_1^e,

$$e^d = \frac{e_1^e}{1+\sigma_0-\delta}$$

[2] Since domestic investors who are seeking to invest funds in foreign countries represent a minor share of the stock, we do not model an interest parity condition for them.

Deriving the function $\tau(e_0, e^*, r, \sigma_0, \delta)$. At the spot rate, e_0, any agent with expectations e_1^e such that $e_0 \leq e^d$ will seek to enter the market now to convert kina into forex.[3] Given that expectations e_1^e are bounded by $e^* - r$ and $e^* + r$, then the break-even spot rate, e^d, is bounded by $\frac{e^*-r}{1+\sigma_0-\delta}$ and $\frac{e^*+r}{1+\sigma_0-\delta}$. When $e_0 \leq \frac{e^*-r}{1+\sigma_0-\delta}$ all agents will seek to convert kina to forex, that is $\Upsilon = 1$. And when $e_0 > \frac{e^*+r}{1+\sigma_0-\delta}$ no agents will seek to convert kina now, that is $\Upsilon = 0$. Given that all agents hold an equal share of the stock of kina, and given that the marginal distribution for exchange rate expectations is given by (1), then at spot rate e_0 the fraction of the kina stock entering the market is $1 - \int_{\frac{e^*-r}{1+\sigma_0-\delta}}^{e_0} f_e((1+\sigma_0-\delta)e, e^*, r)de$. Using the cumulative distribution function, we can calculate $\Upsilon(e_0, e^*, r, \sigma_0, \delta) = 1 - F_e((1+\sigma_0-\delta)\, e_0, e^*, r)$ as follows:

$$\Upsilon(e_0, e^*, r, \sigma_0, \delta) = 1 - \frac{2}{\pi r^2} \int_{\frac{e^*-r}{1+\sigma_0-\delta}}^{e_0} \left(r^2 - \left((1+\sigma_0-\delta)e - e^*\right)^2\right)^{\frac{1}{2}} de$$

$$\text{for } e_0 \in \left[\frac{e^*-r}{1+\sigma_0-\delta}, \frac{e^*+r}{1+\sigma_0-\delta}\right]$$

This simplifies to:

$$\Upsilon(e_0, e^*, r, \sigma_0, \delta) = -\frac{1}{2} + \frac{1}{\pi}\left[\begin{array}{c} sin^{-1}\left(\frac{(1+\sigma_0-\delta)e_0 - e^*}{r}\right) + \\ \left(\frac{(1+\sigma_0-\delta)e_0 - e^*}{r}\right)\left(1 - \left(\frac{(1+\sigma_0-\delta)e_0 - e^*}{r}\right)^2\right)^{\frac{1}{2}} \end{array}\right]$$

$$\text{for } e_0 \in \left[\frac{e^*-r}{1+\sigma_0-\delta}, \frac{e^*+r}{1+\sigma_0-\delta}\right]$$

The properties of $\Upsilon(.)$ may be determined either mathematically or intuitively. First, Υ is decreasing in e_0 so the demand curve, ΥF_d, has a negative slope. An increase in e^* will increase Υ, an increase in r will have an ambiguous effect on Υ, an increase in σ_0 will decrease Υ, and an increase in δ will increase Υ. This may be summarized as follows:

$$\Upsilon\begin{pmatrix} e_0, & e^*, & r, & \sigma_0, & \delta \\ - & + & +/- & - & + \end{pmatrix}$$

C. Supply of Foreign Exchange

Decision of holders of stock of forex. We first derive the shoe-leather cost adjusted interest parity condition. Suppose a foreign investor, or a domestic exporter, holds a stock of foreign exchange, F_s, that they are considering converting into kina at the prevailing spot exchange rate e_0. Converting F_s to kina gives:

$$K_s = e_0 F_S^0$$

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[3] We assume that, when an agent is indifferent between converting or delaying, that is when $e_0 = \frac{1}{1+\sigma_0-\delta}$, then the agents choose to convert now.

where K_s is the equivalent holdings in kina. The investor will receive rate of return over the period of the investment of i leading to $(1 + i)\, K_s$ which they must then convert back into forex in the next period at the exchange rate prevailing at that time, e_1. However, to convert kina into forex, they may also incur the shoe-leather costs, σ_1. As noted above, in order to have one kina to convert into forex, an agent must pay $(1 + \sigma_1)$ kina. This leads to foreign exchange in period 1 of:

$$F_S^1 = \frac{(1 + i)\, K_s}{e_1(1 + \sigma_1)}$$

Intuitively, the imposition of shoe-leather costs has the effect of increasing the number kina required to buy a unit of forex, and so has an effect similar to a depreciation of the kina. This can be seen in the expression above by the term $e_1(1 + \sigma_1)$. It should be noted that shoe-leather costs are asymmetric in the sense that they are incurred only in one direction; that is, in the conversion of kina to forex.

Alternatively, investing overseas leads to $(1 + i^*)\, F_S$, at the end of the period, where i^* is the foreign interest return. We assume that $i > i^*$, that is, the rate of return is higher on Papua New Guinea assets. Equating the two investment strategies leads to the shoe-leather cost-adjusted interest parity condition:

$$\frac{1 + i}{1 + i^*} = \frac{e_1(1 + \sigma_1)}{e_0}$$

which simplifies to:[4]

$$\frac{\Delta e^e}{e_0} = \Delta i - \sigma_1$$

where $\Delta i = i - i^*$. This condition ensures that the expected depreciation just offsets the interest rate differential less shoe-leather costs. This expression allows us to calculate the break-even spot rate, e^s, relative to an agent's expected exchange rate, e_1^e, which is:

$$e^s = \frac{e_1^e}{1 + \Delta i - \sigma_1}$$

This may be interpreted as follows: given the current spot rate, e_0, any agent with expectations e_1^e such that $e_0 \leq e^s$ will seek to enter the market now to convert forex into kina.

Deriving the function $\tau(e_0, e^*, r, \Delta i, \sigma_1)$. At the spot rate, e_0, any agent with expectations e_1^e such that $e_0 \geq e^s$ will seek to enter the market now to convert kina into forex.[5] Given that expectations e_1^e are bounded by $e^* - r$ and $e^* + r$, then the break-even spot rate, e^s, is bounded by $\frac{e^*-r}{1+\Delta i-\sigma_1}$ and $\frac{e^*+r}{1+\Delta i-\sigma_1}$. When $e_0 < \frac{e^*-r}{1+\Delta i-\sigma_1}$, then no agents will seek to convert kina to forex, that is $\tau = 0$.

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4 This uses the result that $ln\ (1 + x) \approx x$ when x is small.

5 As above, we assume that when an agent is indifferent between converting or delaying, that is when $e_0 = \frac{e_1^e}{1+\Delta i-\sigma_1}$, then the agents choose to convert now.

And when $e_0 \geq \frac{e^*+r}{1+\Delta i-\sigma_1}$ then all agents will seek to convert kina now, that is $\tau = 1$. As the spot rate increases from the lower bound, $\frac{e^*-r}{1+\Delta i-\sigma_1}$, agents with expectations e_1^e such that $e_0 \geq \frac{e_1^e}{1+\Delta i-\sigma_1}$ will enter the market to convert forex to kina. Given that all agents hold an equal share of the stock of forex, and given (1), then at spot rate e_0 the fraction of the forex stock entering the market is $\int_{\frac{e^*-r}{1+\Delta i-\sigma_1}}^{e_0} f_e\,((1+\Delta i-\sigma)e, e^*, r)de$ which is the cumulative distribution function $F_e\big((1+\Delta i-\sigma)e_0, e^*, r\big)$. Thus we can calculate $\tau(\epsilon, e^*, r, \Delta i, \sigma_1)$ as follows:

$$\tau(e_0, e^*, r, \Delta i, \sigma_1) = \frac{2}{\pi r^2}\int_{\frac{e^*-r}{1+\Delta i-\sigma_1}}^{e_0} \left(r^2 - \left((1+\Delta i-\sigma_1)_{e\,-\,e^*}\right)^2\right)^{\frac{1}{2}} de$$

$$\text{for } e_0 \,\epsilon \left[\frac{e^*-r}{1+\Delta i-\sigma_1}, \frac{e^*+r}{1+\Delta i-\sigma_1}\right]$$

This simplifies to:

$$\tau(e_0, e^*, r, \Delta i, \sigma_1) = \frac{1}{2} + \frac{1}{\pi}\left[\begin{array}{c} sin^{-1}\left(\frac{(1+\Delta i-\sigma_1)e_0 - e^*}{r}\right) + \\ \left(\frac{(1+\Delta i-\sigma_1)e_0 - e^*}{r}\right)\left(1 - \left(\frac{(1+\Delta i-\sigma_1)e_0 - e^*}{r}\right)^2\right)^{\frac{1}{2}} \end{array}\right]$$

$$\text{for } e_0 \,\epsilon \left[\frac{e^*-r}{1+\Delta i-\sigma_1}, \frac{e^*+r}{1+\Delta i-\sigma_1}\right]$$

The properties of $\tau(.)$are as follows. First, τ is increasing in e_0, so the supply curve, τF_S, has a positive slope. Further, an increase in e^* will lower τ, an increase in r will have an ambiguous effect on τ, an increase in Δi will decrease τ, and an increase in σ_1 will increase r. Put more succinctly,

$$\tau\begin{pmatrix} e_0, & e^*, & r, & \Delta i, & \sigma_1 \\ + & - & +/- & - & + \end{pmatrix}$$